Parallel MUS Extraction

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Minimal Unsatisfiable Subformulas (MUSes)

$$\begin{array}{ccc} C_1 & C_2 & C_3 \\ \boxed{(p)} & \boxed{(q)} & \boxed{(\neg p \vee \neg q)} \end{array}$$

$M = \{C_1, C_2, C_3\}$ is UNSAT

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$$\underset{\boxed{(p)}}{C_1} \quad \underset{\boxed{(q)}}{C_2} \quad \underset{\boxed{(\neg p \vee \neg q)}}{C_3}$$

$M = \{C_1, C_2, C_3\}$ is UNSAT, and $\forall C \in M$, $M \setminus \{C\}$ is SAT.

Minimal Unsatisfiable Subformulas (MUSes)

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$M = \{C_1, C_2, C_3\}$ is *minimal unsatisfiable (MU)*.

Minimal Unsatisfiable Subformulas (MUSes)

C_1	C_2	C_3	C_4	C_5	C_6
(p)	(q)	$(\neg p \vee \neg q)$	$(\neg p \vee r)$	$(p \vee q)$	$(\neg q \vee \neg r)$

$M = \{C_1, C_2, C_3\}$ is *minimal unsatisfiable (MU)*.
$F = \{C_1, \ldots, C_6\}$ is UNSAT, but not MU.

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Applications

Identification and repair of sources of inconsistency:

- circuit error diagnosis; error localization in product configuration

Identification of important/relevant features of systems:

- automatic abstraction in model checking
- environmental assumptions in formal equivalence checking

Complexity Decision: D^P-complete. Function: $\in FP^{NP}$

MUS Extraction

Based on detection of *necessary* (or, *transition*) clauses:

- $C \in F$ is *necessary* for F if $F \in$ UNSAT and $F \setminus \{C\} \in$ SAT.
- If C is necessary for F, then C is in every MUS of F.

MUS Extraction

Based on detection of *necessary* (or, *transition*) clauses:

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- If C is necessary for F, then C is in every MUS of F.

```
Input ↦ Output: F ∈ UNSAT ↦ M ∈ MUS(F)
⟨F_w, M⟩ ← ⟨F, ∅⟩                    // Working formula, MUS under-approx.
while M ≠ F_w do                      // Inv:  M ⊆ F, and ∀C ∈ M is nec. for F_w
    C ← PickClause(F_w)
    st = SAT(F_w \ {C})                          // Test if C is nec. for F_w
    if st = true then                            // If SAT, C is nec. for F_w
        M ← M ∪ {C}
        RMR(F_w, M, τ)          // Model rotation: find more nec. clauses
    else
        F_w ← F_w \ {C}
return M                                                        // M ∈ MUS(F)
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- Hybrid MUS extraction algorithm [Marques-Silva&Lynce'11]

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Based on detection of *necessary* (or, *transition*) clauses:

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Input $\mapsto$ **Output**: $F \in \text{UNSAT} \mapsto M \in \text{MUS}(F)$

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MUS Extraction: opportunities for parallelization

Input $\mapsto$ **Output**: $F \in \mathsf{UNSAT} \mapsto M \in \mathsf{MUS}(F)$
$\langle F_w, M\rangle \leftarrow \langle F, \emptyset\rangle$ `// Working formula, MUS under-approx.`
while $M \neq F_w$ **do** `// Inv:` $M \subseteq F$, `and` $\forall C \in M$ `is nec. for` F_w
    $C \leftarrow \texttt{PickClause}(F_w)$
    $(\mathsf{st}, U, \tau) = \texttt{SAT}(F_w \setminus \{C\})$ `//` U `- unsat. core,` τ `- model`
    if st = true **then** `// If SAT,` C `is nec. for` F_w
        $M \leftarrow M \cup \{C\}$
        $\texttt{RMR}(F_w, M, \tau)$ `// Model rotation: find more nec. clauses`
    else
        $F_w \leftarrow U$ `// Clause-set refinement: discard non-core clauses`
return M `//` $M \in \mathsf{MUS}(F)$

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MUS Extraction: opportunities for parallelization

1. Parallelize each SAT call

Input $\mapsto$ **Output**: $F \in \text{UNSAT} \mapsto M \in \text{MUS}(F)$
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2. Parallelize the main loop, i.e. test multiple clauses

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MUS Extraction: opportunities for parallelization

1. Parallelize each SAT call
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3. Parallel portfolio of MUS extractors

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MUS Extraction: opportunities for parallelization

1. Parallelize each SAT call
2. Parallelize the main loop, i.e. test multiple clauses $\leftarrow$ this talk/paper
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while $M \neq F_w$ **do** // Inv: $M \subseteq F$, and $\forall C \in M$ is nec. for F_w
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Parallelizing the main loop

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while $M \neq F_w$ **do**
  $\{C_1, C_2\} \leftarrow \texttt{PickClauses}(F_w)$
  $(\mathrm{st}_1, U_1, \tau_1) = \texttt{SAT}(F_w \setminus \{C_1\}) \quad || \quad (\mathrm{st}_2, U_2, \tau_2) = \texttt{SAT}(F_w \setminus \{C_2\})$
  sleepUntilFinished() // Wait for both threads to finish
  if $\mathrm{st}_1 = \mathrm{true}$ **and** $\mathrm{st}_2 = \mathrm{true}$ **then**
    $M \leftarrow M \cup \{C_1, C_2\}$
    $\texttt{RMR}(F_w, M, \tau_1)$; $\texttt{RMR}(F_w, M, \tau_2)$
  else if $\mathrm{st}_1 = \mathrm{true}$ **and** $\mathrm{st}_2 = \mathrm{false}$ **then**
    $M \leftarrow M \cup \{C_1\}$
    $\texttt{RMR}(F_w, M, \tau_1)$; $F_w \leftarrow U_2$
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while M ≠ F_w do
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    (st_1, U_1, τ_1) = SAT(F_w \ {C_1})  ||  (st_2, U_2, τ_2) = SAT(F_w \ {C_2})
    sleepUntilFinished()                            // Wait for both threads to finish
    if st_1 = true and st_2 = true then
        M ← M ∪ {C_1, C_2}
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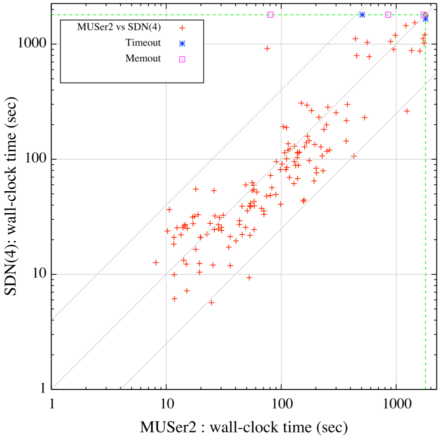
Parallelizing the main loop



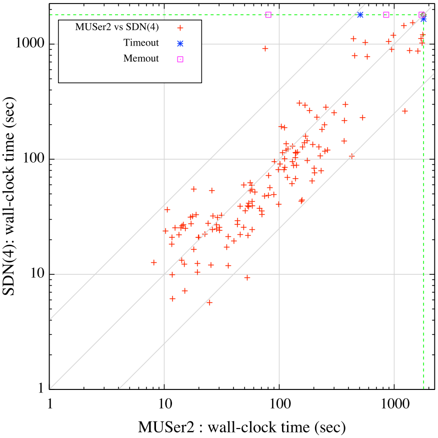
175 benchs, MUS track, SC'11.

wall-clock limit 1800 sec

memory limit 16 GB.

	#sol.	avg.time
(x) sequential	144	186.46
(y) parallel, 4 thr.	143	154.93

Parallelizing the main loop



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Shortcomings

(i) Threads are under-utilized because of synchronization.

(ii) No *communication*, i.e. exchange of learned clauses between threads.

Parallelizing the main loop: de-synchronizing

Technicalities

"Outdated" SAT outcomes are OK — if C is necessary for F_w, it is also necessary for $F'_w \subset F_w$.

"Outdated" UNSAT cores might be not — test if $U \subseteq F_w$, if not drop it.

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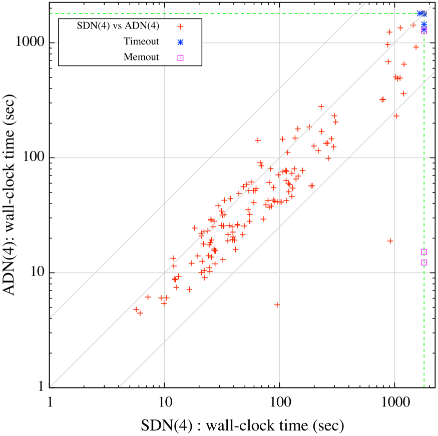

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	#sol.	avg.time
(x) parallel, 4 thr. synchronous	143	154.93
(y) parallel, 4 thr. asynchronous	146	126.45

Parallelizing the main loop: communication

Would like to exchange clauses between threads

Problem: threads work on *different* formulas $\rightarrow$ clauses learned by one might be not valid for another.

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$$F \quad \overset{C_1}{\boxed{(p)}} \; \overset{C_2}{\boxed{(q)}} \; \overset{C_3}{\boxed{(\neg p \vee \neg q)}}$$

Thread 1: solves $SAT(F \setminus \{C_1\})$, derives $(\neg p)$.
Thread 2: works on $SAT(F \setminus \{C_2\})$, receives $(\neg p)$, returns UNSAT.

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Thread 1: solves $SAT(F \setminus \{C_1\})$, derives $(\neg p)$.
Thread 2: works on $SAT(F \setminus \{C_2\})$, receives $(\neg p)$, returns UNSAT.

Solution: assumption-based, incremental SAT [Eén, Sörensson, ENTCS 2003]

Note: most modern MUS extractors use assumption-based incremental SAT anyway.

Assumption-based incremental SAT [Eén, Sörensson, ENTCS 2003]

SAT solver interface

add($\{C_1, \ldots, C_n\}$) — add clauses $C_1, \ldots, C_n$ to the SAT solver.

solve($\{l_1, \ldots, l_k\}$) — determine the satisfiability of the current set of clauses under a partial assignment defined by literals $\{l_1, \ldots, l_k\}$.

Assumption-based incremental SAT [Eén, Sörensson, ENTCS 2003]

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$$F \qquad \overset{C_1}{\boxed{(p)}}\ \overset{C_2}{\boxed{(q)}}\ \overset{C_3}{\boxed{(\neg p \vee \neg q)}}\ \overset{C_4}{\boxed{(p \vee q)}}$$

$$F_A \qquad \overset{C_1}{\boxed{(a_1 \vee p)}}\ \overset{C_2}{\boxed{(a_2 \vee q)}}\ \overset{C_3}{\boxed{(a_3 \vee \neg p \vee \neg q)}}\ \overset{C_4}{\boxed{(a_4 \vee p \vee q)}}$$

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	C_1	C_2	C_3	C_4
F	(p)	(q)	$(\neg p \vee \neg q)$	$(p \vee q)$
F_A	$(a_1 \vee p)$	$(a_2 \vee q)$	$(a_3 \vee \neg p \vee \neg q)$	$(a_4 \vee p \vee q)$

To test $F \setminus \{C_1\}$: add(F_A); solve($\{a_1, \neg a_2, \neg a_3, \neg a_4\}$) $\rightarrow$ SAT, model

To test $F \setminus \{C_4\}$: add(F_A); solve($\{\neg a_1, \neg a_2, \neg a_3, a_4\}$) $\rightarrow$ UNSAT, core

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F_A	$\boxed{(a_1 \vee p)}$	$\boxed{(a_2 \vee q)}$	$\boxed{(a_3 \vee \neg p \vee \neg q)}$	$\boxed{(a_4 \vee p \vee q)}$

To test $F \setminus \{C_1\}$: add(F_A); solve($\{a_1, \neg a_2, \neg a_3, \neg a_4\}$) $\rightarrow$ SAT, model

To test $F \setminus \{C_4\}$: add(F_A); solve($\{\neg a_1, \neg a_2, \neg a_3, a_4\}$) $\rightarrow$ UNSAT, core

Note: learned clauses are entailed by *input* clauses — can be exchanged.

Assumption-based incremental SAT [Eén, Sörensson, ENTCS 2003]

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	C_1	C_2	C_3	C_4
F	(p)	(q)	$(\neg p \vee \neg q)$	$(p \vee q)$
F_A	$(a_1 \vee p)$	$(a_2 \vee q)$	$(a_3 \vee \neg p \vee \neg q)$	$(a_4 \vee p \vee q)$

To test $F \setminus \{C_1\}$: add(F_A); solve($\{a_1, \neg a_2, \neg a_3, \neg a_4\}$) $\rightarrow$ SAT, model

To test $F \setminus \{C_4\}$: add(F_A); solve($\{\neg a_1, \neg a_2, \neg a_3, a_4\}$) $\rightarrow$ UNSAT, core

Note: learned clauses are entailed by *input* clauses — can be exchanged.

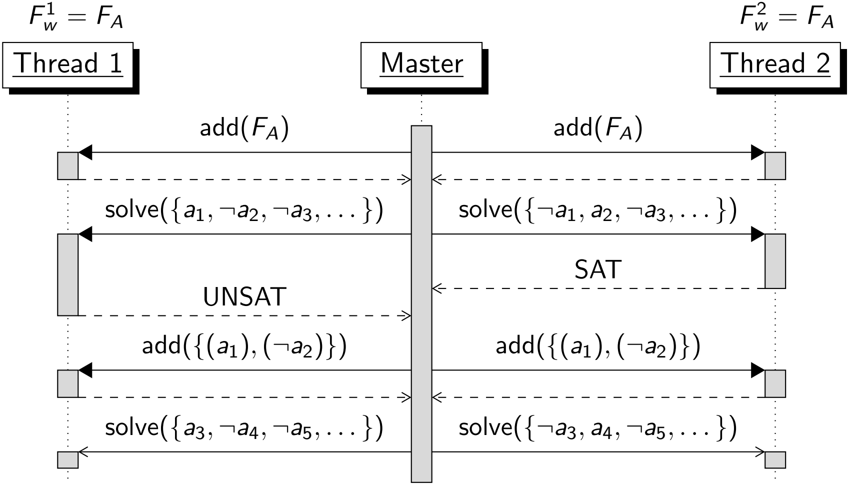
To "remove" C_4 from F_A : add({ (a_4) }).

To *finalize* C_1 in F_A : add({ $(\neg a_1)$ }).

Note: there is another approach [Marques-Silva, Sakallah, FTCS 1997; Nadel, Ryvchin, SAT 2012]

Incremental SAT and Parallel MUS Extraction (sync)

$$F_A = \{(a_1 \vee C_1), (a_2 \vee C_2), (a_3 \vee C_3), (a_4 \vee C_4), \dots\}$$



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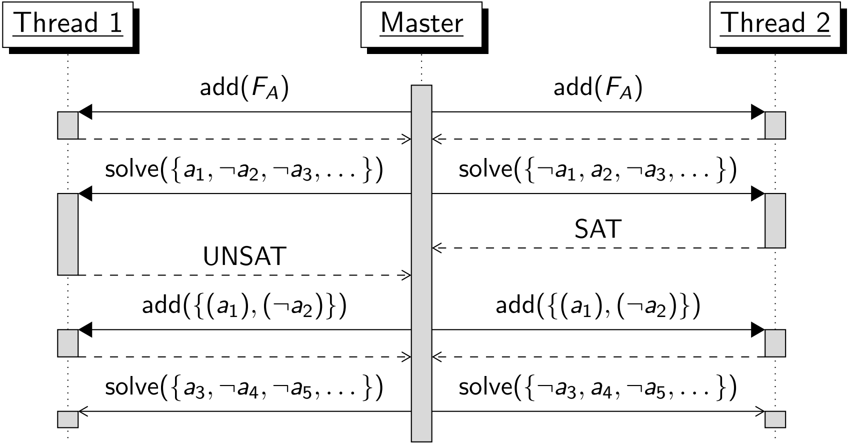
$$F_A = \{(a_1 \vee C_1), (a_2 \vee C_2), (a_3 \vee C_3), (a_4 \vee C_4), \dots\}$$

$F_w^1 = F_A \cup \{(a_1), (\neg a_2)\}$

$F_w^2 = F_A \cup \{(a_1), (\neg a_2)\}$



Incremental SAT and Parallel MUS Extraction (sync)

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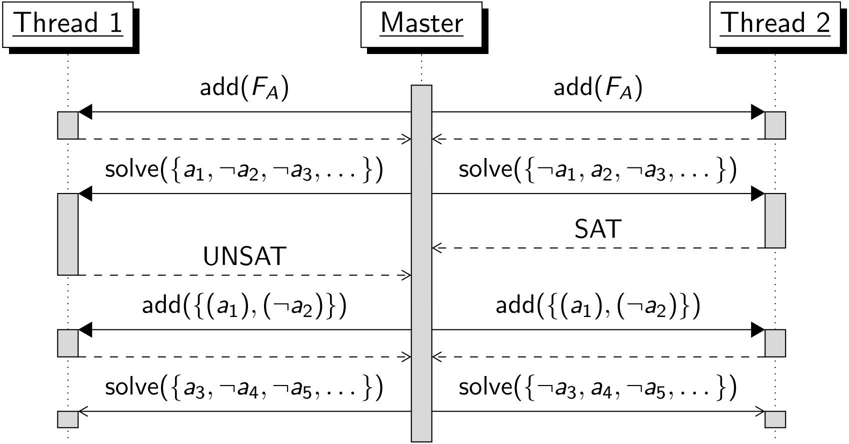
$F_w^1 = F_A \cup \{(a_1), (\neg a_2)\}$ $F_w^2 = F_A \cup \{(a_1), (\neg a_2)\}$



Threads always work on the *same* formula $\rightarrow$ unrestricted clause exchange.

Incremental SAT and Parallel MUS Extraction (async)

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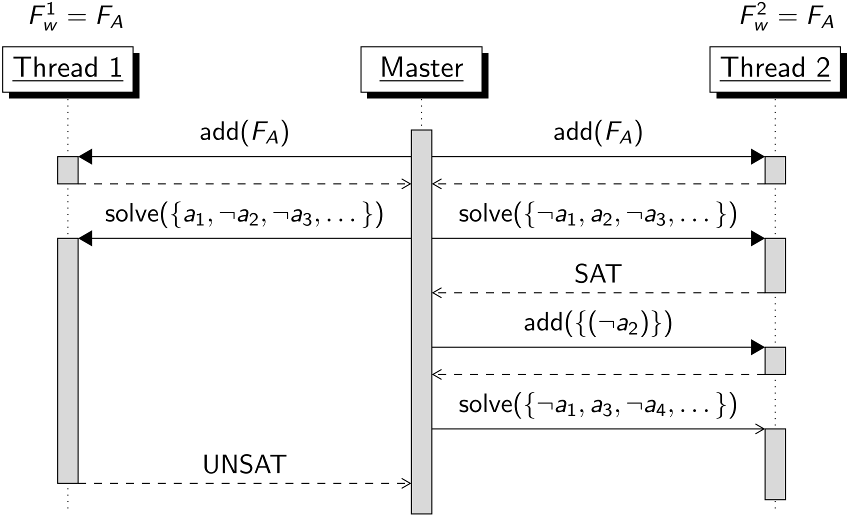

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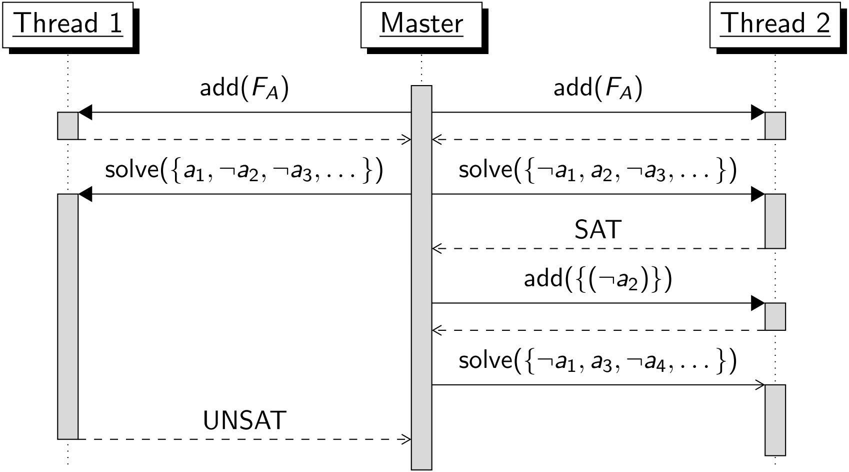

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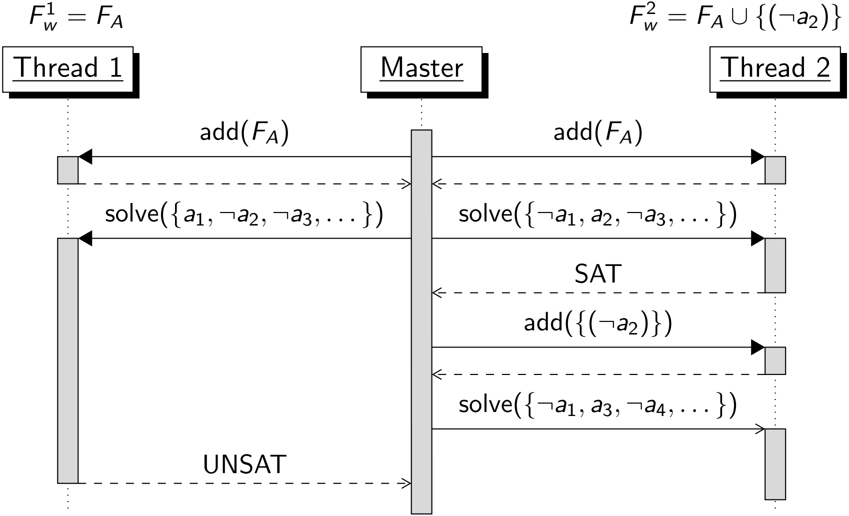

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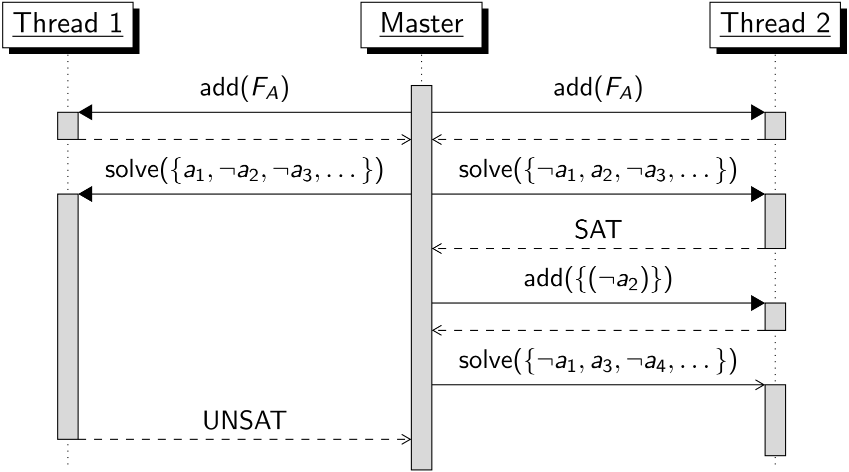

Threads work on different formulas: Thread 2 $\rightarrow$ Thread 1 ?

Soundness of “back” communication

$$F_A = \{(a_1 \vee C_1), (a_2 \vee C_2), (a_3 \vee C_3), (a_4 \vee C_4), \dots\}$$

Thread 1 (“behind”): $F_w^1 = F_A$, solve$(\{a_1, \neg a_2, \neg a_3, \dots\})$

Thread 2 (“ahead”): $F_w^2 = F_A \cup \{(\neg a_2)\}$, solve$(\{\neg a_1, a_3, \neg a_4, \dots\})$

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C – a clause learned by Thread 2. We have $F_A \cup \{(\neg a_2)\} \vDash C$.

C is not entailed by F_A, but since Thread 1 is solving under assumption $\neg a_2$, it is valid for the duration of the call.

Before the next call $(\neg a_2)$ will be added to Thread 1 by the Master, and C will be again entailed by the input clauses.

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Since a_2 appears only positively in F_A, no clause with a_2 will participate in the conflict. So, $F_A \vDash C$, and C can be used by Thread 1.

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Bottom line: *unrestricted* communication is possible — due to the assumption-based incremental SAT.

Improving communication

Would like to exchange *promising* clauses only.

- Restrict clause size (def: ≤ 10)
- Restrict clause LBD (def: ≤ 5)
- Optionally: change the limits dynamically
- Initialize ("bump") activity of received clauses.

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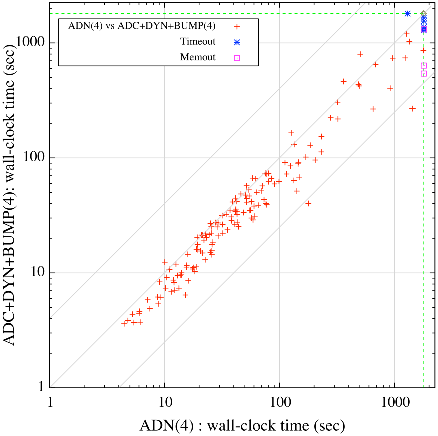
Note: a good idea for non-parallel MUS extraction as well [Audemard, Lagniez, Simon, SAT 2013] (tomorrow morning).

Parallelizing the main loop: communication



175 benchs, MUS track, SC'11.

wall-clock limit 1800 sec

memory limit 16 GB.

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(x) parallel, 4 thr. no communication	146	126.45
(y) parallel, 4 thr. communication	153	133.98

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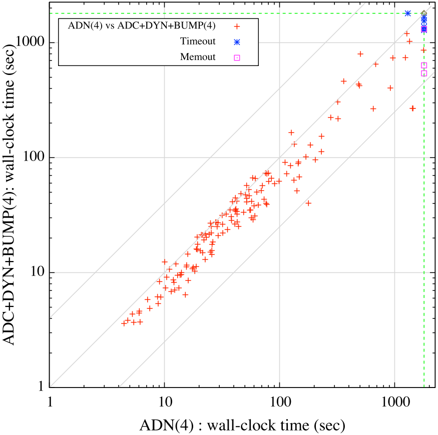

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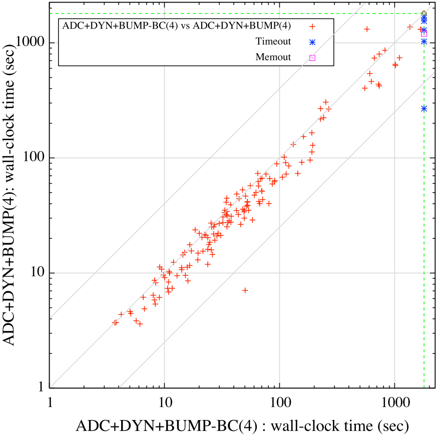
Communication is essential for performance.

Sound communication is enabled by *incremental SAT*.

Note: interestingly, sound resolution-based preprocessing for MUS extraction is also enabled by incremental SAT [Belov, Järvisalo, Marques-Silva, TACAS 2013]

Impact of “back” communication



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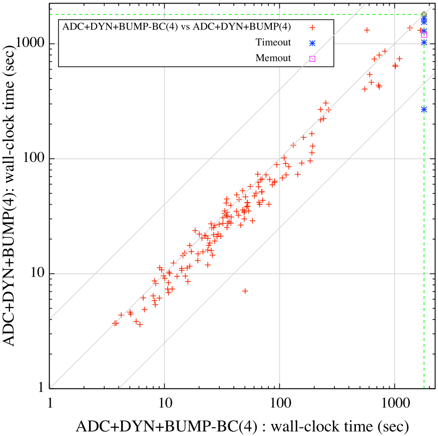

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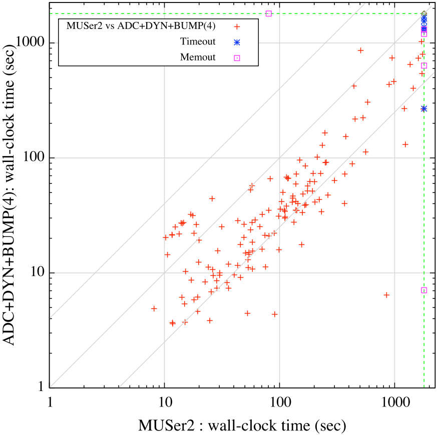
“Back” communication is actually quite crucial.

Parallelizing the main loop: communication



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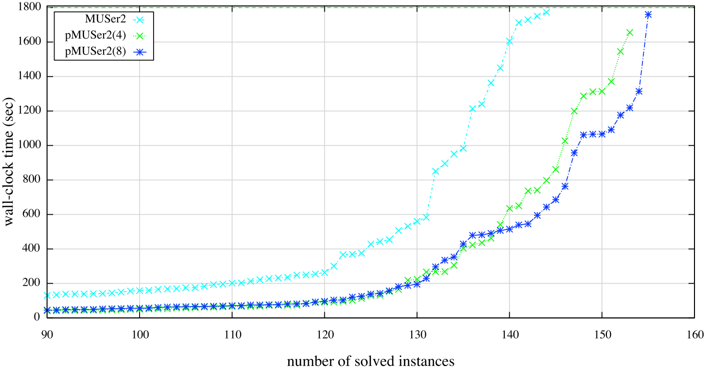
	#sol.	avg.time
(x) sequential	144	186.46
(y) parallel, 4 thr. async. + comm.	153	133.98

Performance and scalability from 4 to 8 cores



	Min. speedup	Avg. sp.	Max. sp.	Med. sp.
Seq. vs 4 cores	0.49x	4.09x	132.59x	2.94x
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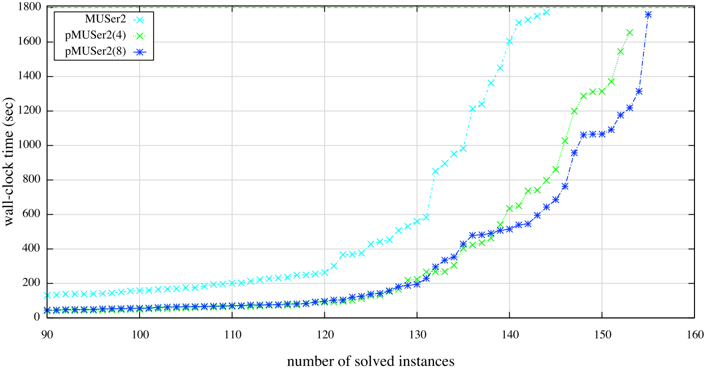

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Possible reasons: (*i*) duplication of work; (*ii*) parallelization overhead on easy SAT calls.

Final Remarks

Also in the paper ...

- "Core-based" scheduling — a slight improvement on 8 cores.
- Results for group-MUS — less exciting than for plain-MUS.
- Comparison with TarmoMUS [Wieringa, CP 2012 and Wieringa, Heljanko, TACAS 2013] ... see the paper ☺

Main points

- Incremental SAT is a key technology for for enabling efficient parallel MUS extraction.
- Assumptions should be treated as superfluous during clause exchange.
- Good scalability to 4 cores; but not 8. Possible approaches:
  - A good partitioning/job distribution heuristic.
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Thank you for your attention !