Upper and Lower Bounds for Weak Backdoor Set Detection

Neeldhara Misra, **Sebastian Ordyniak**, Venkatesh Raman, and Stefan Szeider

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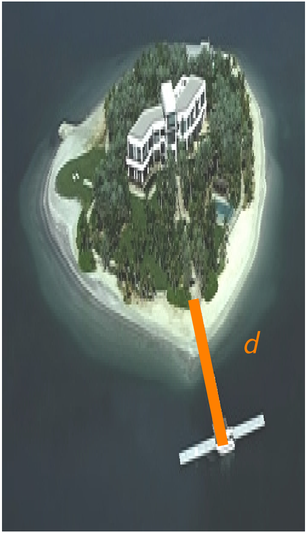
Backdoor Sets

- Introduced by Crama et al. 1997 and independently by Williams et al. 2003 in an attempt to explain the good performance of SAT-solvers.
- Have been intensively studied as a structural parameter in various fields of AI (Gaspers and Szeider 2012).
- Provide a measure for the distance of a CNF-formula to some tractable base class.



Weak Backdoor Sets

Definition

Let $\mathcal{C}$ be a tractable class of CNF formulas, F a CNF formula, and B a set of variables of F. Then B is a **weak $\mathcal{C}$-backdoor set** of F if there is an assignment τ for the variables of B such that: $F[\tau]$ is satisfiable and $F[\tau] \in \mathcal{C}$.

Observation

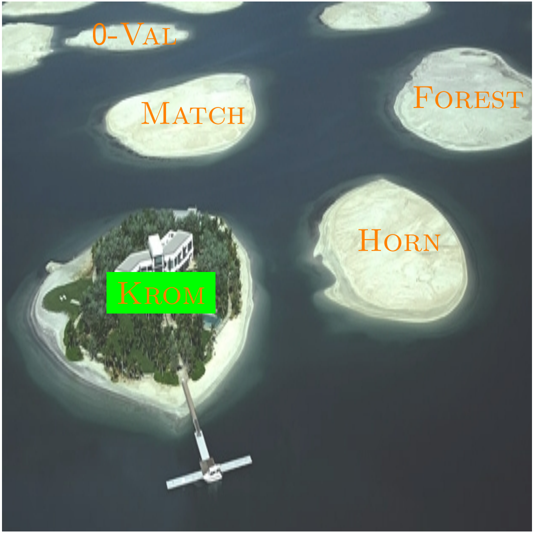
Given a formula F and a weak $\mathcal{C}$-backdoor set B for some tractable class $\mathcal{C}$, then a satisfying assignment of F can be found in time $O(2^{|B|}p(|F|))$.

Hence, the main task is to efficiently find a small weak backdoor set!

Islands of Tractability

We consider the following "islands of tractability":

- KROM
- HORN and CO-HORN
- 0-VAL and 1-VAL
- FOREST
- MATCH



Complexity of Finding Weak Backdoor Sets

- Unfortunately, for all of these base classes , finding weak backdoor sets cannot be done efficiently, i.e., it is fixed-parameter intractable!
- However, if we restrict the length of the clauses of the input formula to a constant, then finding weak backdoor sets is fixed-parameter tractable (for all but MATCH).

Here we focus on exact upper bounds and lower bounds for the complexity of finding a weak backdoor set when the input formula has at most 3 literals per clause ($3\mathrm{CNF}$).

Finding Weak Backdoor Sets

We consider the following problem (here $\mathcal{C}$ is a tractable class of CNF formulas):

$\textsc{Weak } (3\mathrm{CNF}, \mathcal{C})\textsc{-Backdoor Detection}$ **Parameter:** k

Input: A formula $3\mathrm{CNF}$ formula F and a natural number k.
Question: Does F have a weak $\mathcal{C}$-backdoor set of size at most k?

Our Results

- We improve the current upper bounds for weak backdoor detection for the classes Krom and Horn from 6^k to 2.27^k and 4.54^k, respectively.
- We show the first lower bounds for weak backdoor detection for the classes Krom, Horn, 0-Val, Forest, and Match.

Our Results – in detail

Upper bounds and lower bounds for WEAK $(3\mathrm{CNF}, \mathcal{B})$-BACKDOOR DETECTION:

$\mathcal{B}$	Lower bound	Upper bound
KROM	2^k	2.27^k
HORN	2^k	4.54^k
0-VAL	$2^{o(k)}$	2.85^k (1)
FOREST	2^k	$f(k)$ (2)
MATCH	$n^{\frac{k}{2}-\epsilon}$	n^k

(1) Raman and Shankar 2013

(2) Gaspers and Szeider 2012

(3) Gaspers, Ordyniak, Ramanujan, Saurabh, and Szeider 2013

Methods

Lower bounds

We show the lower bounds by a reduction from SAT using the (Strong) Exponential Time Hypothesis.

Upper bounds

- The algorithm for KROM uses a reduction to 3-HITTING SET.
- For HORN we use a sophisticated branching algorithm applying ideas from Raman and Shankar 2013.

Conclusion

We initiated a systematic study of the complexity of finding weak backdoor sets of $3\mathrm{CNF}$ formulas. This lead to:

- improved algorithms for several base classes, and
- the first lower bounds for many base classes.

Future Work

- Close the gaps between upper and lower bounds of the considered problems.
- Study Weak $(\mathcal{A}, \mathcal{B})$-backdoor Set for other restrictions of the input formulas ($\mathcal{A}$) than $3\mathrm{CNF}$.

Thank You!