Soundness of Inprocessing in Clause Sharing SAT Solvers[1]

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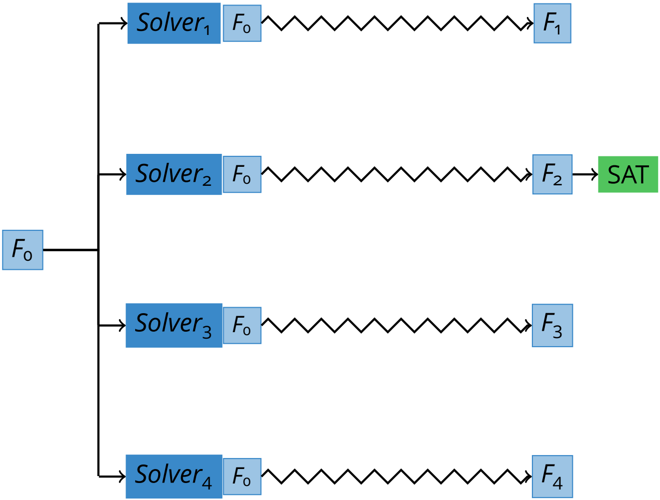
[1]The second author was supported by the International Master's Program in Computational Logic (MCL)

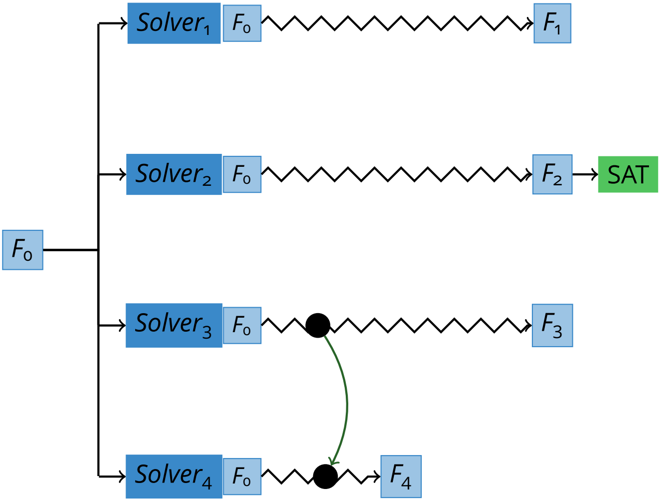
Outline

(1) Formalisms for Clause Sharing SAT Solvers with Inprocessing
(2) A Novel Way to Combine Clause Addition Techniques
(3) Conclusion

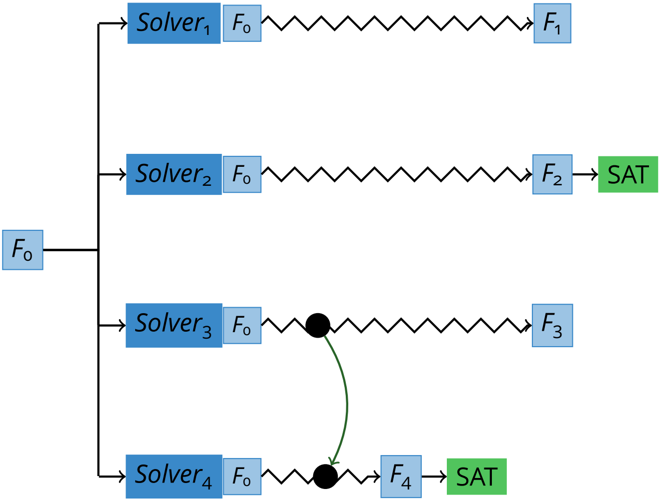
The Portfolio Approach



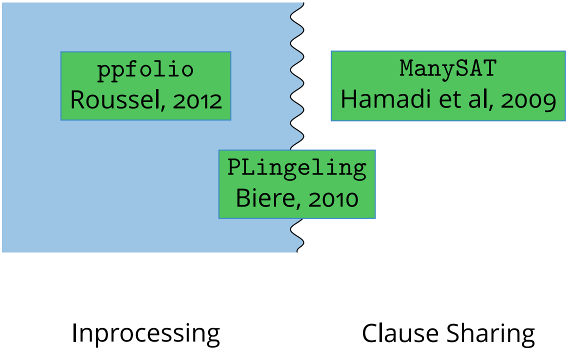
The Cooperative Portfolio Approach



The Cooperative Portfolio Approach



Inprocessing in Clause Sharing SAT Solvers



Portfolios can be Described as State Transition Systems

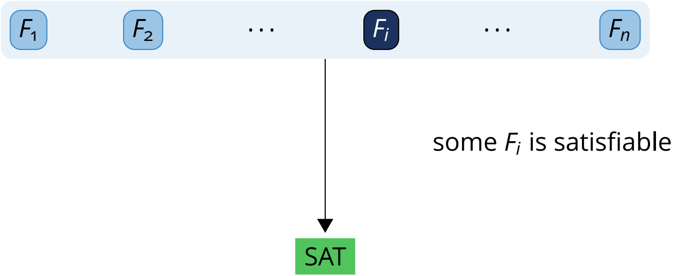
- **Local state for** *Solver*$_i$: F_i
- **State with multiplicity** n: $(F_1, \ldots, F_n)$, SAT, UNSAT
- **Initial state for** F_0 **with multiplicity** n: $\mathrm{init}(n, F_0) = (F_0, \ldots, F_0)$
- **Final states**: SAT, UNSAT
- **Transition relation**: $\rightsquigarrow$
- **Soundness**:
  (i) if $\mathrm{init}(n, F_0) \overset{*}{\rightsquigarrow}$ SAT, then F_0 is satisfiable, and
  (ii) if $\mathrm{init}(n, F_0) \overset{*}{\rightsquigarrow}$ UNSAT, then F_0 is unsatisfiable

System A

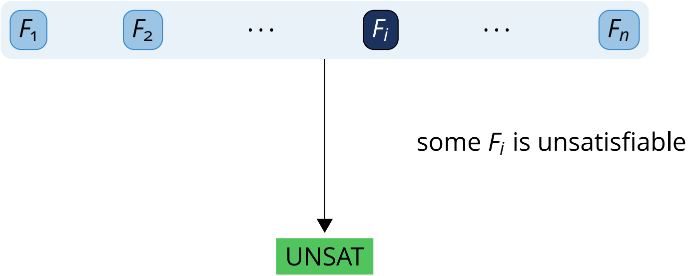
Where Equivalence is Preserved

SAT, UNSAT, CM, CS

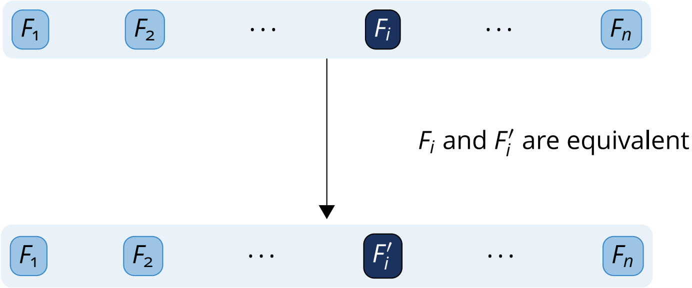
SAT Termination Rule



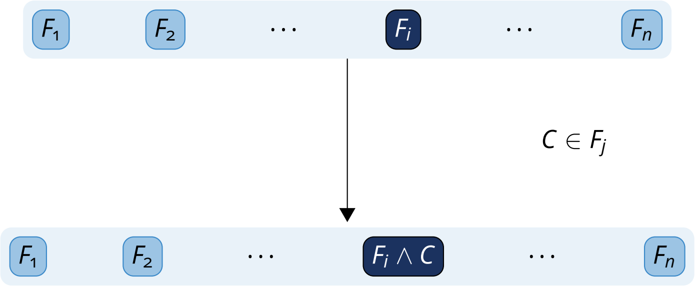
UNSAT Termination Rule



Clause Management Rule



Clause Sharing Rule



Properties of System A

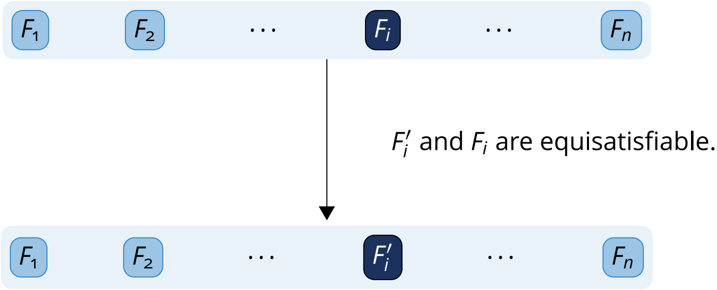
- **Equivalence-Preserving:**
  - Probing
  - Hyper Binary Resolution
- **Instances**: ManySAT, Penelope
- **Key Invariant:** If $\mathrm{init}(n, F_0) \overset{*}{\rightsquigarrow} (F_1, \ldots, F_n)$, then:
  - $F_i \equiv F_0$ for all $i \in \{1, \ldots, n\}$.
- **Theorem**: System A is sound.

System B

Inprocessing without Limits

SAT, UNSAT, CM, CS, UI

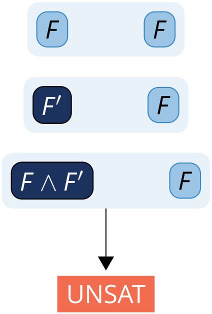
Unrestricted Inprocessing Rule



- F' **and** F **are equisatisfiable iff**:
  (i) F and F' are satisfiable, or
  (ii) F and F' are unsatisfiable

UNSAT can be Incorrect: Unrestricted Inprocessing and Clause Sharing.

- $F = (x), F' = (\overline{x})$
- F and F' are equisatisfiable
- $F \wedge F'$ is unsatisfiable.



Properties of System B

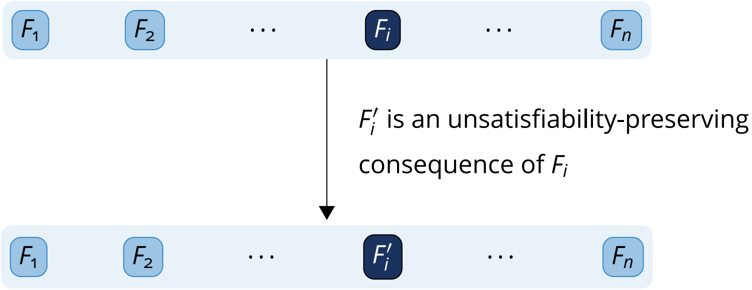
- **Satisfiability-Preserving**:
  - Variable Elimination
  - Equivalence Elimination
  - Blocked Clause Elimination and Addition
  - Extended Resolution
  - Bounded Variable Addition
- **Instances:** ?
- **Theorem:**
  - System B is sound w.r.t. SAT
  - System B is unsound w.r.t. UNSAT.

System C

With Clause Deletion Techniques

SAT, UNSAT, CM, CS, RI, ER, BVA

Restricted Inprocessing Rule

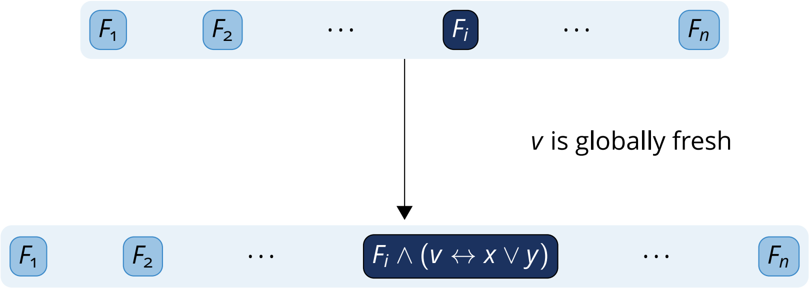


- F' **is an unsatisfiability-preserving consequence of** F:
  (i) $F \models F'$, and
  (ii) if F is unsatisfiable, then F' is unsatisfiable.

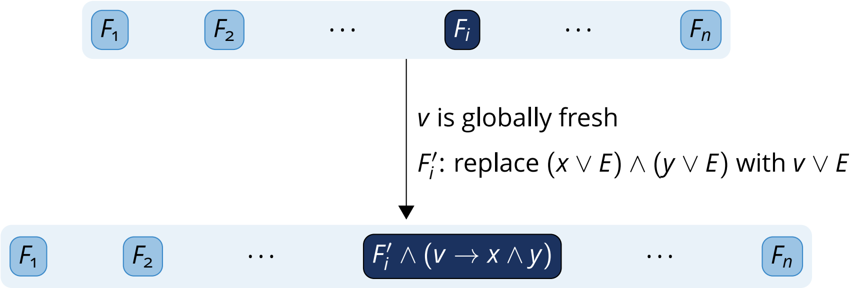
Many Simplification Methods Produce Unsatisfiability-Preserving Consequences

- Blocked Clause Elimination
- Variable Elimination
- Equivalence Elimination

Extended Resolution Rule



Bounded Variable Addition Rule



Properties of System C

- **Unsatisfiability-Preserving Consequences**:
  - Blocked Clause Elimination
  - Equivalence Elimination
  - Variable Elimination
  - Equivalence-Preserving Techniques
- **Instances:** PLingeling
- **Key Invariant:** If $\text{init}(n, F_0) \overset{*}{\leadsto} (F_1, \ldots, F_n)$, then:
  - (i) $F_0 \wedge D \models F_1 \wedge \ldots \wedge F_n$
  - (ii) F_0 and $F_0 \wedge D$ are equisatisfiable
  - (iii) F_i and F_0 are equisatisfiable for all $i \in \{1, \ldots, n\}$.
- **Theorem:** System C is sound.

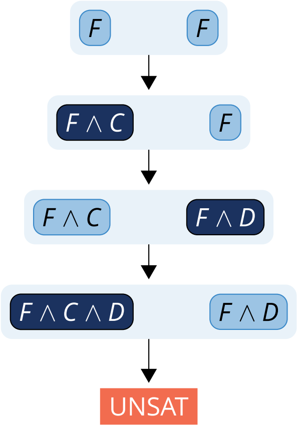
System D

With Clause Addition Techniques

SAT, UNSAT, CM, CS, ER, BVA, ADD, DEL

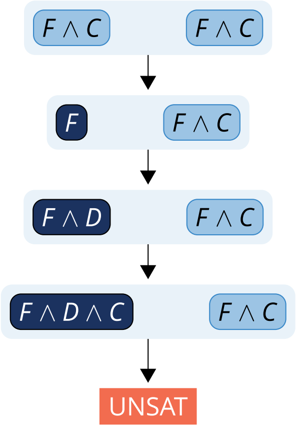
UNSAT can be Incorrect: Applying Clause Addition Techniques in Two Solvers

- $F = (x \vee \overline{y}) \wedge (\overline{x} \vee y) \wedge (x \vee z) \wedge (y \vee \overline{z})$
- $C = (\overline{x} \vee \overline{z})$ is blocked in F by $\overline{z}$
- $D = (\overline{y} \vee z)$ is blocked in F by z
- F is satisfiable:
  - $I = \{x, y, z\}$
  - $J = \{x, y, \overline{z}\}$
- $I \not\models C, J \not\models D$
- $F \wedge C \wedge D$ is unsatisfiable

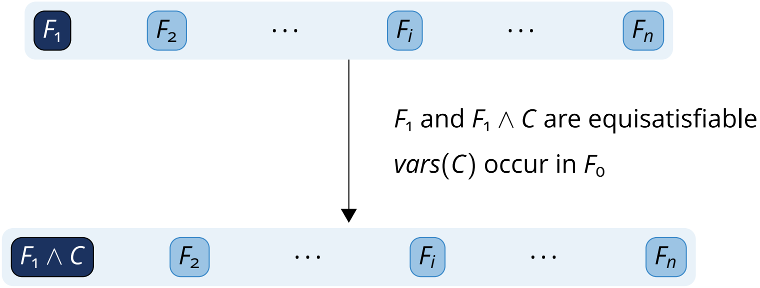


UNSAT can be Incorrect: Applying Clause Elimination and Addition Techniques in One Solver

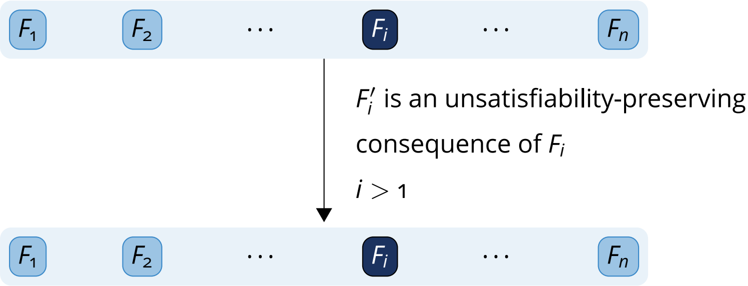
- $F = (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (x \vee z) \wedge (y \vee \bar{z})$
- $C = (\bar{x} \vee \bar{z})$ is blocked in F by $\bar{z}$
- $D = (\bar{y} \vee z)$ is blocked in F by z
- F is satisfiable:
  - $I = \{x, y, z\}$
  - $J = \{x, y, \bar{z}\}$
- $F \wedge C$ is satisfiable:
  - $J = \{x, y, z\}$
- $F \wedge C \wedge D$ is unsatisfiable, since $J \not\models D$



Clause Addition Rule



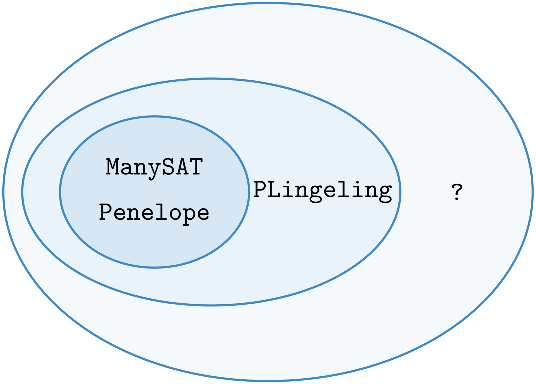
Clause Deletion Rule



Properties of System D

- **Clause Addition and Deletion Techniques:**
  - Unsatisfiability-Preserving Consequences
  - Blocked Clause Addition
- **Key Invariant:** If $\text{init}(n, F_0) \overset{*}{\rightsquigarrow} (F_1, \ldots, F_n)$, then:
  (i) $F_1 \wedge D \models F_2 \wedge \ldots \wedge F_n$
  (ii) F_1 and $F_1 \wedge D$ are equisatisfiable
  (iii) F_i and F_0 are equisatisfiable for all $i \in \{1, \ldots, n\}$.
- **Theorem**: System D is sound.

Conclusion



- **Unsatisfiability-Preserving Consequences**

Future Work

(1) Can portfolios be improved by adding a single distinguished solver incarnation that performs clause addition techniques?

(2) How can we extend the formalisms to parallel solvers based on the search-space splitting approach?

(3) How can we extend the formalisms to parallel solvers with multiple input formulas?

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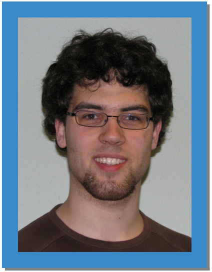
Acknowledgement



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